

Расчетные задания

Задача 1. Довести, что $\lim_{n \rightarrow \infty} a_n = a$ (указать $N(\varepsilon)$).

1. $a_n = \frac{3n-2}{2n-1}, a = \frac{3}{2}$.

3. $a_n = \frac{7n+4}{2n+1}, a = \frac{7}{2}$.

5. $a_n = \frac{7n-1}{n+1}, a = 7$.

7. $a_n = \frac{9-n^3}{1+2n^3}, a = -\frac{1}{2}$.

9. $a_n = \frac{1-2n^2}{2+4n^2}, a = -\frac{1}{2}$.

11. $a_n = \frac{n+1}{1-2n}, a = -\frac{1}{2}$.

13. $a_n = \frac{1-2n^2}{n^2+3}, a = -2$.

15. $a_n = \frac{n}{3n-1}, a = \frac{1}{3}$.

17. $a_n = \frac{4+2n}{1-3n}, a = -\frac{2}{3}$.

19. $a_n = \frac{3-n^2}{1+2n^2}, a = -\frac{1}{2}$.

21. $a_n = \frac{3n-1}{5n+1}, a = \frac{3}{5}$.

23. $a_n = \frac{1-2n^2}{2+4n^2}, a = -\frac{1}{2}$.

25. $a_n = \frac{2-2n}{3+4n}, a = -\frac{1}{2}$.

27. $a_n = \frac{1+3n}{6-n}, a = -3$.

29. $a_n = \frac{3n^2+2}{4n^2-1}, a = \frac{3}{4}$.

2. $a_n = \frac{4n-1}{2n+1}, a = 2$.

4. $a_n = \frac{2n-5}{3n+1}, a = \frac{2}{3}$.

6. $a_n = \frac{4n^2+1}{3n^2+2}, a = \frac{4}{3}$.

8. $a_n = \frac{4n-3}{2n+1}, a = 2$.

10. $a_n = -\frac{5n}{n+1}, a = -5$.

12. $a_n = \frac{2n+1}{3n-5}, a = \frac{2}{3}$.

14. $a_n = \frac{3n^2}{2-n^2}, a = -3$.

16. $a_n = \frac{3n^3}{n^3-1}, a = 3$.

18. $a_n = \frac{5n+15}{6-n}, a = -5$.

20. $a_n = \frac{2n-1}{2-3n}, a = -\frac{2}{3}$.

22. $a_n = \frac{4n-3}{2n+1}, a = 2$.

24. $a_n = \frac{5n+1}{10n-3}, a = \frac{1}{2}$.

26. $a_n = \frac{23-4n}{2-n}, a = 4$.

28. $a_n = \frac{2n+3}{n+5}, a = 2$.

30. $a_n = \frac{2-3n^2}{4+5n^2}, a = -\frac{3}{5}$.

Задача 2. Застосовуючи приклад 5, §1, обчислити границі числових послідовностей.

$$1. \lim_{n \rightarrow \infty} \frac{(3-n)^2 + (3+n)^2}{(3-n)^2 - (3+n)^2}.$$

$$3. \lim_{n \rightarrow \infty} \frac{(3-n)^4 - (2-n)^4}{(1-n)^3 - (1+n)^3}.$$

$$5. \lim_{n \rightarrow \infty} \frac{(6-n)^2 - (6+n)^2}{(6+n)^2 - (1-n)^2}.$$

$$7. \lim_{n \rightarrow \infty} \frac{(1+2n)^3 - 8n^3}{(1+2n)^2 + 4n^2}.$$

$$9. \lim_{n \rightarrow \infty} \frac{(3-n)^3}{(n+1)^2 - (n+1)^3}.$$

$$11. \lim_{n \rightarrow \infty} \frac{2(n+1)^3 - (n-2)^3}{n^2 + 2n - 3}.$$

$$13. \lim_{n \rightarrow \infty} \frac{(n+3)^3 + (n+4)^3}{(n+3)^4 - (n+4)^4}.$$

$$15. \lim_{n \rightarrow \infty} \frac{8n^3 - 2n}{(n+1)^4 - (n-1)^4}.$$

$$17. \lim_{n \rightarrow \infty} \frac{(2n-3)^3 - (n+5)^3}{(3n-1)^3 + (2n+3)^3}.$$

$$19. \lim_{n \rightarrow \infty} \frac{(2n+1)^3 + (3n+2)^3}{(2n+3)^3 - (n-7)^3}.$$

$$21. \lim_{n \rightarrow \infty} \frac{(2n+1)^3 - (2n+3)^3}{(2n+1)^2 + (2n+3)^2}.$$

$$23. \lim_{n \rightarrow \infty} \frac{(n+2)^4 - (n-2)^4}{(n+5)^2 + (n-5)^2}.$$

$$25. \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 - (n-1)^2}.$$

$$27. \lim_{n \rightarrow \infty} \frac{(n+2)^3 + (n-2)^3}{n^4 + 2n^2 - 1}.$$

$$2. \lim_{n \rightarrow \infty} \frac{(3-n)^4 - (2-n)^4}{(1-n)^4 - (1+n)^4}.$$

$$4. \lim_{n \rightarrow \infty} \frac{(1-n)^4 - (1+n)^4}{(1+n)^3 - (1-n)^3}.$$

$$6. \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n+1)^2}{(n-1)^3 - (n+1)^3}.$$

$$8. \lim_{n \rightarrow \infty} \frac{(3-4n)^2}{(n-3)^3 - (n+3)^3}.$$

$$10. \lim_{n \rightarrow \infty} \frac{(n+1)^2 + (n-1)^2 - (n+2)^3}{(4-n)^3}.$$

$$12. \lim_{n \rightarrow \infty} \frac{(n+1)^3 + (n+2)^3}{(n+4)^3 + (n+5)^3}.$$

$$14. \lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^3 + (n-1)^3}.$$

$$16. \lim_{n \rightarrow \infty} \frac{(n+6)^3 - (n+1)^3}{(2n+3)^2 + (n+4)^2}.$$

$$18. \lim_{n \rightarrow \infty} \frac{(n+10)^2 + (3n+1)^2}{(n+6)^3 - (n+1)^3}.$$

$$20. \lim_{n \rightarrow \infty} \frac{(n+7)^3 - (n+2)^3}{(3n+2)^2 + (4n+1)^2}.$$

$$22. \lim_{n \rightarrow \infty} \frac{n^3 - (n-1)^3}{(n+1)^4 - n^4}.$$

$$24. \lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^3 + (n-1)^3}.$$

$$26. \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2}.$$

$$28. \lim_{n \rightarrow \infty} \frac{(n+1)^3 + (n-1)^3}{n^3 - 3n}.$$

$$29. \lim_{n \rightarrow \infty} \frac{(n+1)^3 + (n-1)^3}{n^3 + 1}.$$

$$30. \lim_{n \rightarrow \infty} \frac{(n+2)^2 - (n-2)^2}{(n+3)^2}.$$

Задача 3. Застосовуючи приклад 6, §1, обчислити границі числових послідовностей.

$$1. \lim_{n \rightarrow \infty} \frac{n \sqrt[3]{5n^2} + \sqrt[4]{9n^8 + 1}}{(n + \sqrt{n}) \sqrt{7 - n + n^2}}.$$

$$2. \lim_{n \rightarrow \infty} \frac{\sqrt{n-1} - \sqrt{n^2 + 1}}{\sqrt[3]{3n^3 + 3} + \sqrt[4]{n^5 + 1}}.$$

$$3. \lim_{n \rightarrow \infty} \frac{\sqrt{n^3 + 1} - \sqrt{n-1}}{\sqrt[3]{n^3 + 1} - \sqrt{n-1}}.$$

$$4. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 - 1} + 7n^3}{\sqrt[4]{n^{12} + n + 1} - n}.$$

$$5. \lim_{n \rightarrow \infty} \frac{\sqrt{3n-1} - \sqrt[3]{125n^3 + n}}{\sqrt[5]{n} - n}.$$

$$6. \lim_{n \rightarrow \infty} \frac{n \sqrt[5]{n} - \sqrt[3]{27n^6 + n^2}}{(n + \sqrt[4]{n}) \sqrt{9 + n^2}}.$$

$$7. \lim_{n \rightarrow \infty} \frac{\sqrt{n+2} - \sqrt{n^2 + 2}}{\sqrt[4]{4n^4 + 1} - \sqrt[3]{n^4 - 1}}.$$

$$8. \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + 2} + \sqrt{n-2}}{\sqrt[4]{n^4 + 2} + \sqrt{n-2}}.$$

$$9. \lim_{n \rightarrow \infty} \frac{6n^3 - \sqrt{n^5 + 1}}{\sqrt{4n^6 + 3} - n}.$$

$$10. \lim_{n \rightarrow \infty} \frac{\sqrt{5n+2} - \sqrt[3]{8n^3 + 5}}{\sqrt[4]{n+7} - n}.$$

$$11. \lim_{n \rightarrow \infty} \frac{n \sqrt[4]{3n+1} + \sqrt{81n^4 - n^2 + 1}}{(n + \sqrt[3]{n}) \sqrt{5 - n + n^2}}.$$

$$12. \lim_{n \rightarrow \infty} \frac{\sqrt{n+3} - \sqrt{n^2 - 3}}{\sqrt[3]{n^5 - 4} - \sqrt[4]{n^4 + 1}}.$$

$$13. \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + 3} - \sqrt{n-3}}{\sqrt[5]{n^5 + 3} + \sqrt{n-3}}.$$

$$14. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} - 9n^2}{3n - \sqrt[4]{9n^8 + 1}}.$$

$$15. \lim_{n \rightarrow \infty} \frac{\sqrt{4n+1} - \sqrt[3]{27n^3 + 4}}{\sqrt[4]{n} - \sqrt[3]{n^5 + n}}.$$

$$16. \lim_{n \rightarrow \infty} \frac{n \sqrt[3]{7n} - \sqrt[4]{81n^8 - 1}}{(n + 4\sqrt{n}) \sqrt{n^2 - 5}}.$$

$$17. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^3 - 7} + \sqrt[3]{n^2 + 4}}{\sqrt[4]{n^5 + 5} + \sqrt{n}}.$$

$$18. \lim_{n \rightarrow \infty} \frac{\sqrt{n^6 + 4} + \sqrt{n-4}}{\sqrt[5]{n^6 + 6} - \sqrt{n-6}}.$$

$$19. \lim_{n \rightarrow \infty} \frac{4n^2 - \sqrt[4]{n^3}}{\sqrt[3]{n^6 + n^3 + 1} - 5n}.$$

$$20. \lim_{n \rightarrow \infty} \frac{\sqrt{n+3} - \sqrt[3]{8n^3 + 3}}{\sqrt[4]{n+4} - \sqrt[5]{n^5 + 5}}.$$

$$21. \lim_{n \rightarrow \infty} \frac{n \sqrt[4]{11n} + \sqrt{25n^4 - 81}}{(n - 7\sqrt{n})\sqrt{n^2 - n + 1}}.$$

$$22. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2} - \sqrt{n^2 + 5}}{\sqrt[5]{n^7} - \sqrt{n + 1}}.$$

$$23. \lim_{n \rightarrow \infty} \frac{\sqrt{n^7 + 5} - \sqrt{n - 5}}{\sqrt[7]{n^7 + 5} + \sqrt{n - 5}}.$$

$$24. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 + 2} - 5n^2}{n - \sqrt{n^4 - n + 1}}.$$

$$25. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 2} - \sqrt[3]{n^3 + 2}}{\sqrt[7]{n + 2} - \sqrt[5]{n^5 + 2}}.$$

$$26. \lim_{n \rightarrow \infty} \frac{n \sqrt{71n} - \sqrt[3]{64n^6 + 9}}{(n - \sqrt[3]{n})\sqrt{11 + n^2}}.$$

$$27. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 6} - \sqrt{n^2 - 5}}{\sqrt[3]{n^3 + 3} + \sqrt[4]{n^3 + 1}}.$$

$$28. \lim_{n \rightarrow \infty} \frac{\sqrt{n^8 + 6} - \sqrt{n - 6}}{\sqrt[8]{n^8 + 6} + \sqrt{n - 6}}.$$

$$29. \lim_{n \rightarrow \infty} \frac{n^2 - \sqrt{n^3 + 1}}{\sqrt[3]{n^6 + 2} - n}.$$

$$30. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 1} - \sqrt[3]{n^3 + 1}}{\sqrt[4]{n + 1} - \sqrt[5]{n^5 + 1}}.$$