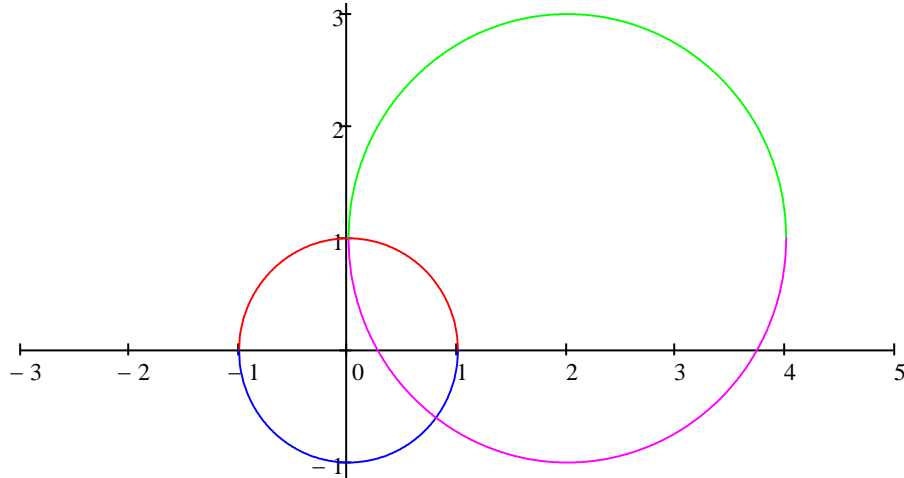


Криві Коло

Задане явно. Верхнє півколо: $y = \sqrt{R^2 - (x - x_0)^2} + y_0$, нижнє півколо: $y = -\sqrt{R^2 - (x - x_0)^2} + y_0$;

— $y = \sqrt{1 - x^2}$, — $y = -\sqrt{1 - x^2}$; — $y = \sqrt{4 - (x - 1)^2} + 2$, — $y = -\sqrt{4 - (x - 1)^2} + 2$.



Задане неявно

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

Параметрично

$$\begin{cases} x = R \cos t + x_0, \\ y = R \sin t + y_0, \end{cases} \quad 0 \leq t < 2\pi.$$

Задане неявно

$$x^2 + y^2 = 1$$

Параметрично

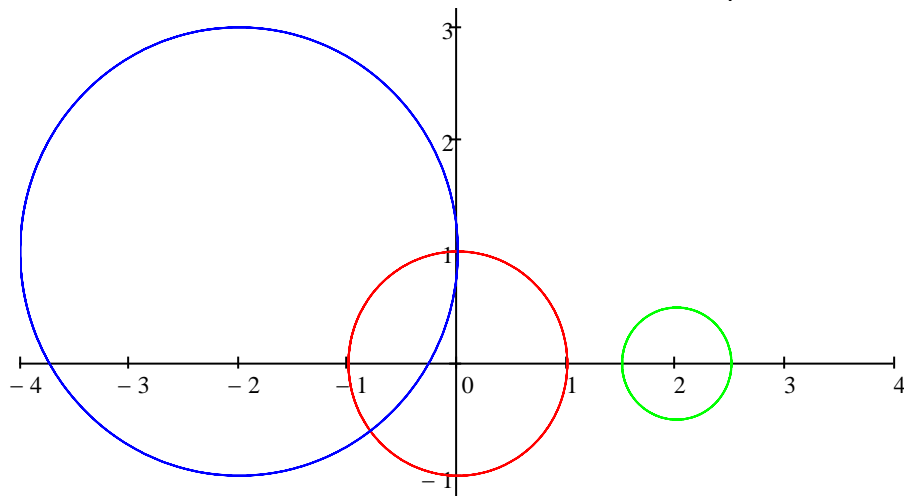
$$\begin{cases} x = \cos t, \\ y = \sin t, \end{cases} \quad 0 \leq t < 2\pi.$$

$$(x + 2)^2 + (y - 1)^2 = 4$$

$$\begin{cases} x = 2 \cos t - 2, \\ y = 2 \sin t + 1, \end{cases} \quad 0 \leq t < 2\pi.$$

$$(x - 2)^2 + y^2 = \frac{1}{4}$$

$$\begin{cases} x = 0.5 \cos t + 2, \\ y = 0.5 \sin t, \end{cases} \quad 0 \leq t < 2\pi.$$



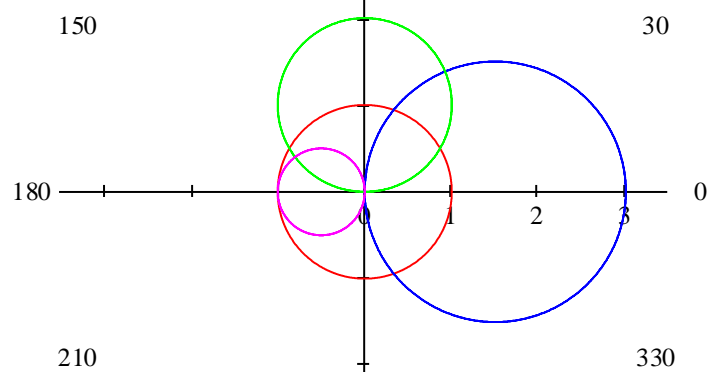
В полярній системі координат

$$\text{— } \rho = 1, \quad 0 \leq \varphi < 2\pi;$$

$$\text{— } \rho = 3 \cos \varphi, \quad -\frac{\pi}{2} \leq \varphi < \frac{\pi}{2};$$

$$\text{— } \rho = 2 \sin \varphi, \quad 0 \leq \varphi < \pi;$$

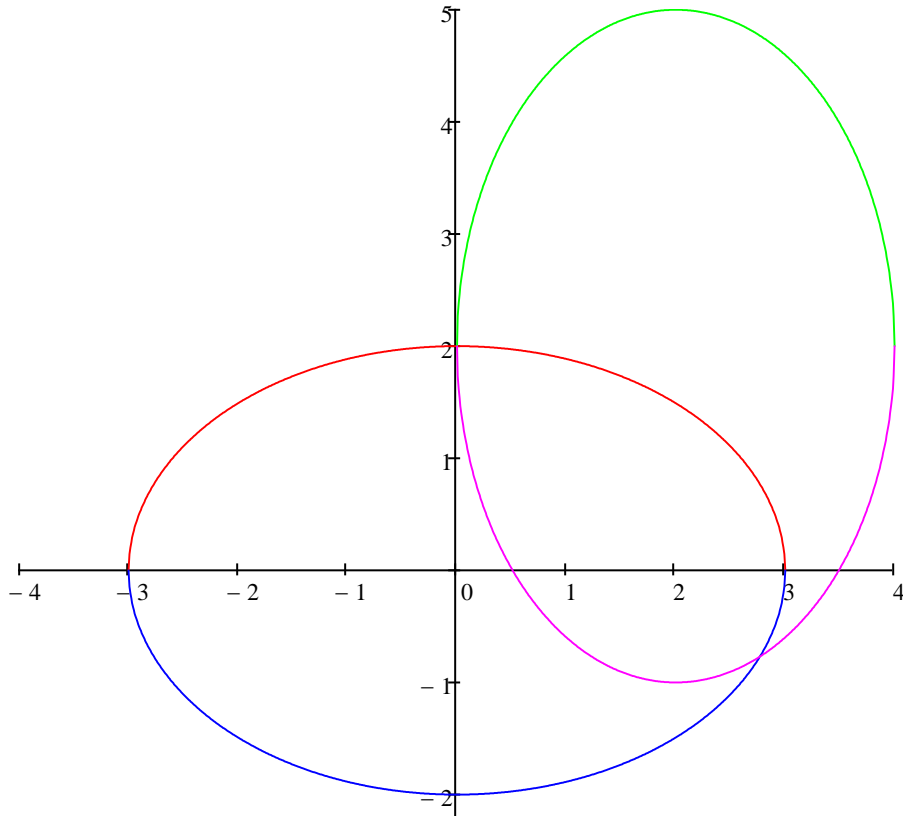
$$\text{— } \rho = -\cos \varphi, \quad \frac{\pi}{2} \leq \varphi < \frac{3\pi}{2}.$$



Еліпс

Заданий явно

$$\text{—} y = 2\sqrt{1 - \frac{x^2}{9}}; \quad \text{—} y = -2\sqrt{1 - \frac{x^2}{9}}; \quad \text{—} y = 3\sqrt{1 - \frac{(x-2)^2}{4}} + 2; \quad \text{—} y = -3\sqrt{1 - \frac{(x-2)^2}{4}} + 2.$$



Заданий неявно

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

$$\text{—} \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\text{—} (x+2)^2 + \frac{(y-1)^2}{4} = 1$$

$$\text{—} \frac{(x-1)^2}{9} + (y+1)^2 = 1$$

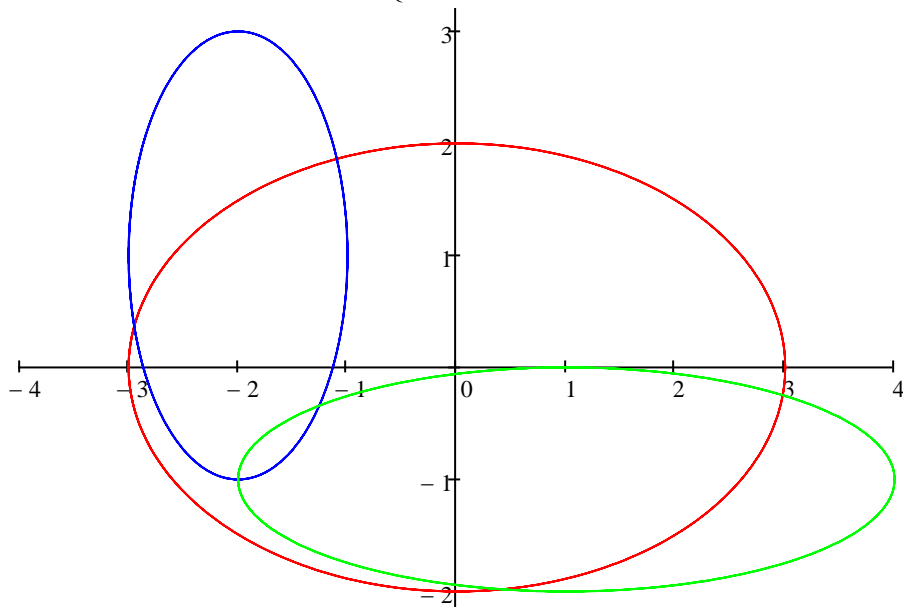
Параметрично

$$\begin{cases} x = a \cos t + x_0, \\ y = b \sin t + y_0, \end{cases} \quad 0 \leq t < 2\pi.$$

$$\begin{cases} x = 3 \cos t, \\ y = 2 \sin t, \end{cases} \quad 0 \leq t < 2\pi.$$

$$\begin{cases} x = \cos t - 2, \\ y = 2 \sin t + 1, \end{cases} \quad 0 \leq t < 2\pi.$$

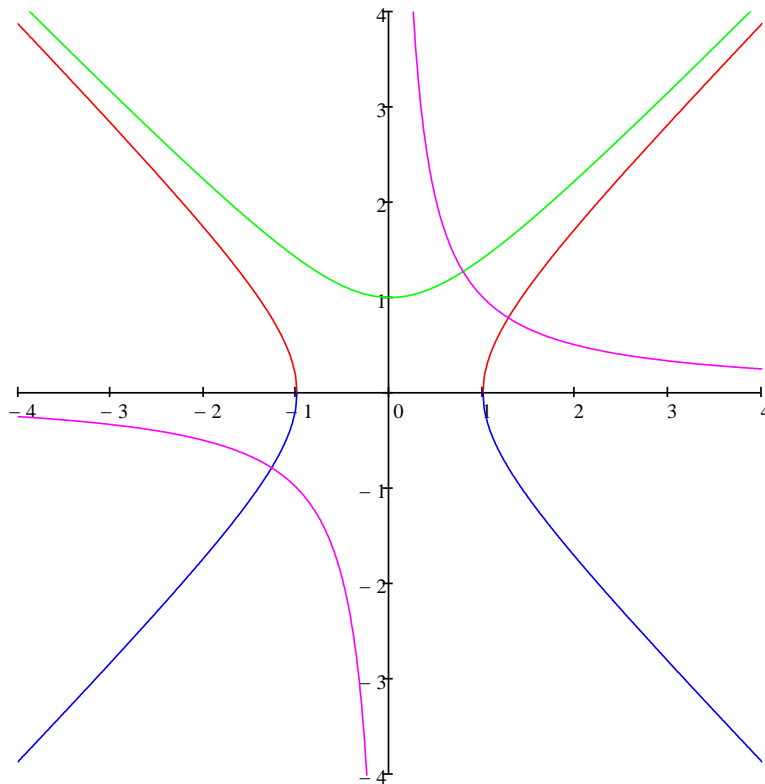
$$\begin{cases} x = 3 \cos t + 1, \\ y = \sin t - 1, \end{cases} \quad 0 \leq t < 2\pi.$$



Гіпербола

Задана явно

— $y = \sqrt{x^2 - 1}$; — $y = -\sqrt{x^2 - 1}$; — $y = \sqrt{x^2 + 1}$; — $y = \frac{1}{x}$.



Задана неявно

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1$$

— $\frac{(x + 2)^2}{4} - \frac{(y - 1)^2}{4} = 1$

права гілка

Параметрично
(одна гілка)

$$\begin{cases} x = \pm a \operatorname{ch} t + x_0, \\ y = b \operatorname{sh} t + y_0. \end{cases}$$

$$\begin{cases} x = 2 \operatorname{ch} t - 2, \\ y = 2 \operatorname{sh} t + 1. \end{cases}$$

Задана неявно

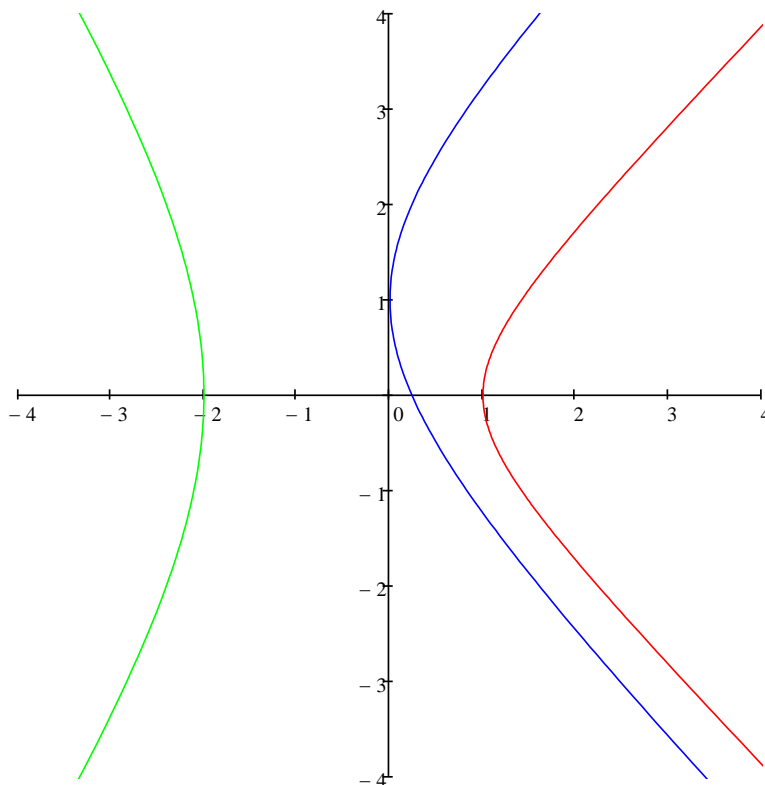
— $x^2 - y^2 = 1$ права гілка

— $\frac{x^2}{4} - \frac{y^2}{9} = 1$ ліва гілка

Параметрично
(одна гілка)

$$\begin{cases} x = \operatorname{ch} t, \\ y = \operatorname{sh} t. \end{cases}$$

$$\begin{cases} x = -2 \operatorname{ch} t, \\ y = 3 \operatorname{sh} t. \end{cases}$$



Парабола

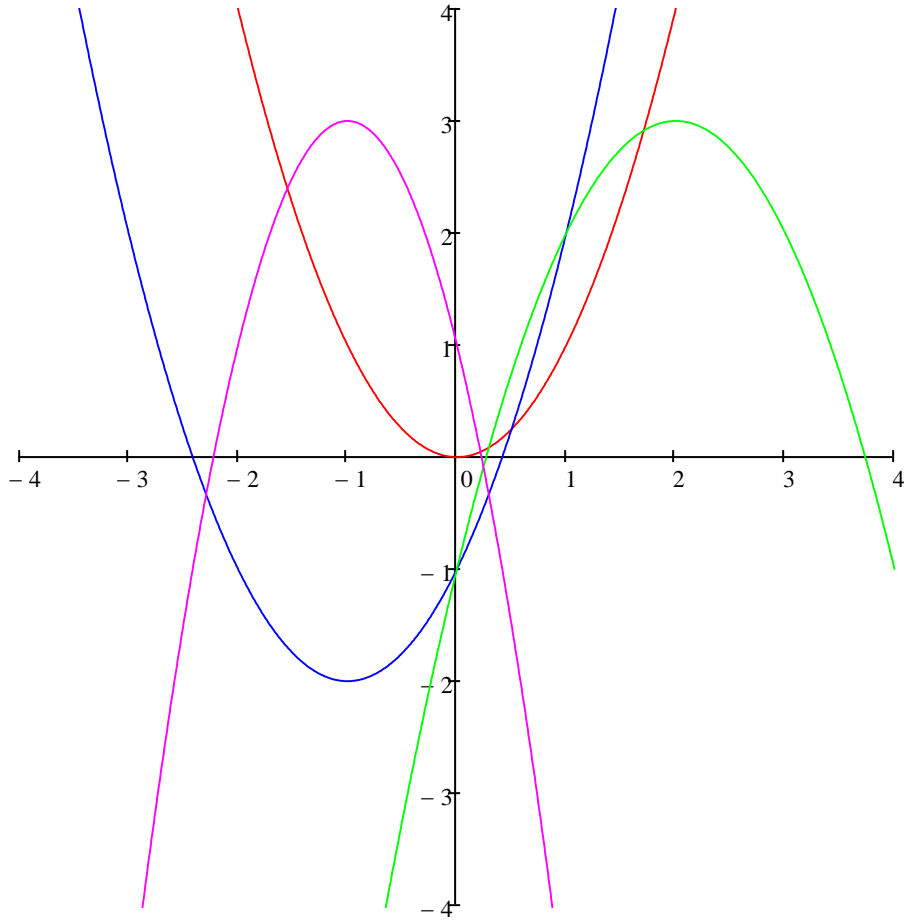
$$y = a(x - x_0)^2 + y_0$$

— $y = x^2$;

— $y = (x+1)^2 - 2$;

— $y = -(x-2)^2 + 3$;

— $y = -2(x+1)^2 + 3$.

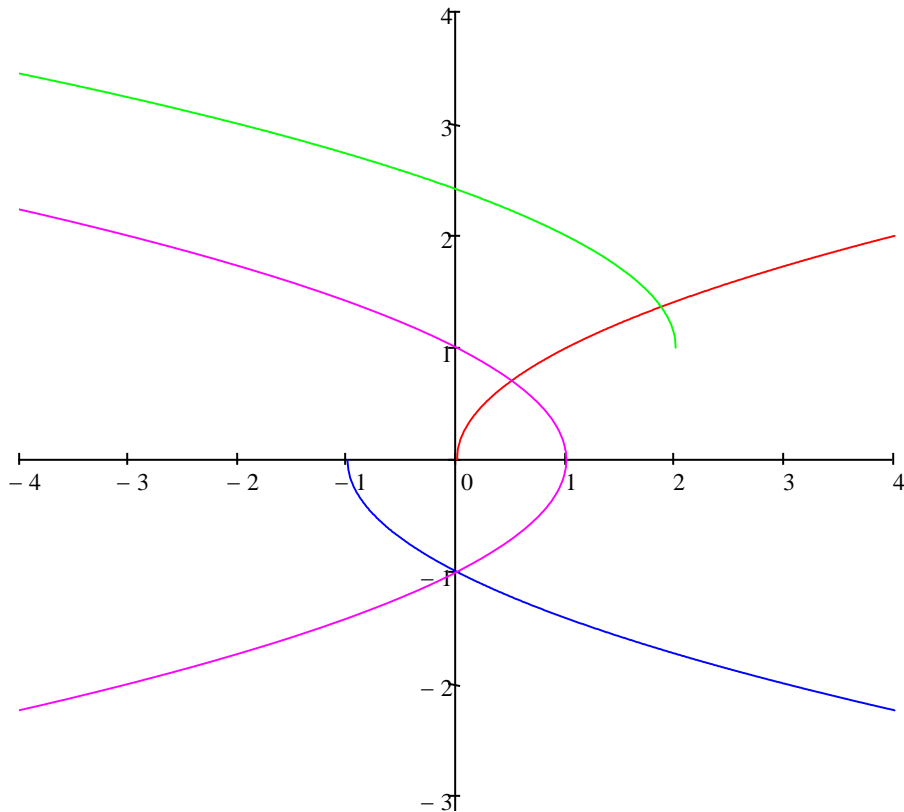


— $y = \sqrt{x}$;

— $y = -\sqrt{x+1}$;

— $y = \sqrt{2-x} + 1$;

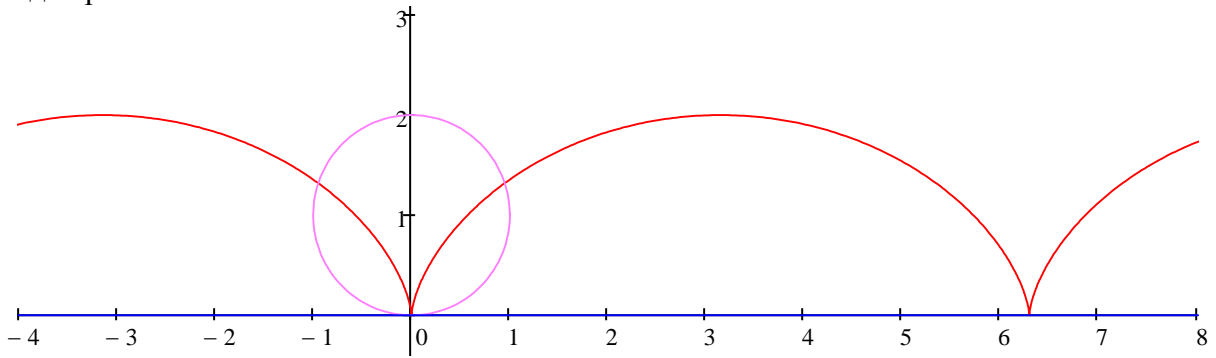
— $x = -y^2 + 1$.



Циклоїда

$$\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t). \end{cases} \quad \text{— рухомий об'єкт (коло); — нерухомий об'єкт (пряма).}$$

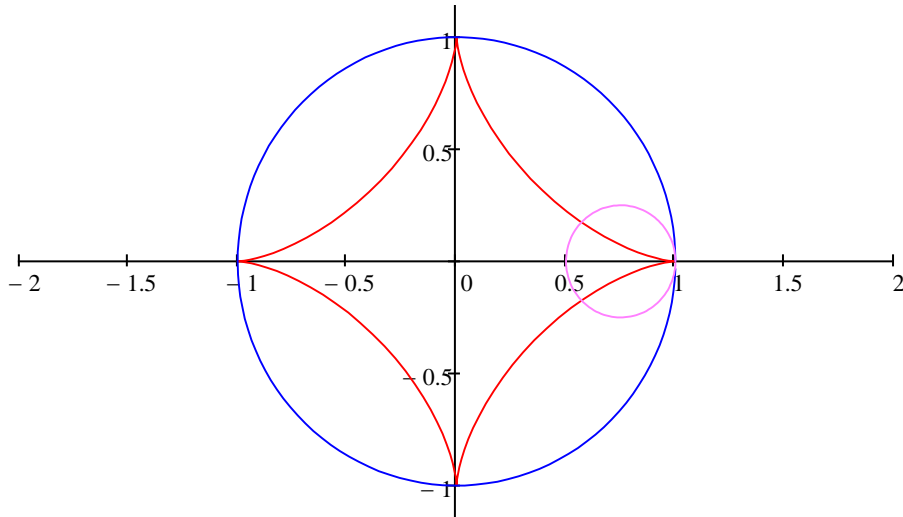
— циклоїда при $a = 1$.



Астроїда

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}, \quad \begin{cases} x = a \cos^3 t, \\ y = a \sin^3 t. \end{cases} \quad \text{— рухомий об'єкт (коло); — нерухомий об'єкт (коло).}$$

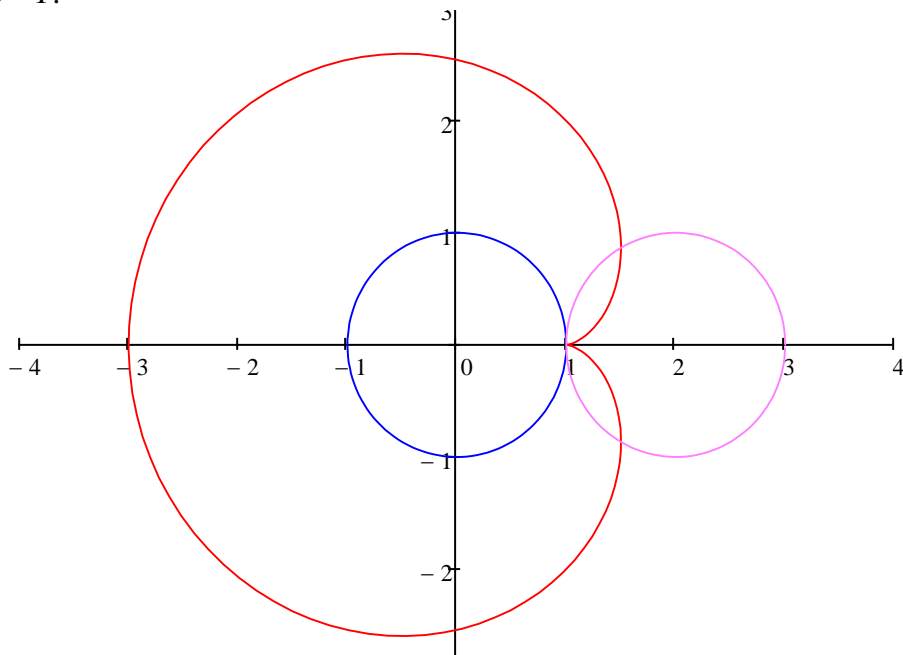
— астроїда при $a = 1$.



Кардіоїда

$$\begin{cases} x = 2a \cos t - a \cos 2t, \\ y = 2a \sin t - a \sin 2t. \end{cases} \quad \text{— рухомий об'єкт (коло); — нерухомий об'єкт (коло).}$$

— кардіоїда при $a = 1$.

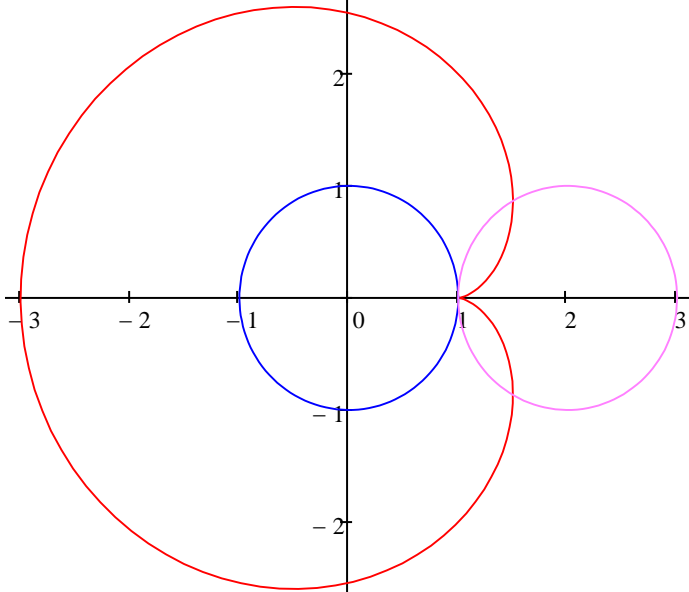


Епіциклоїда

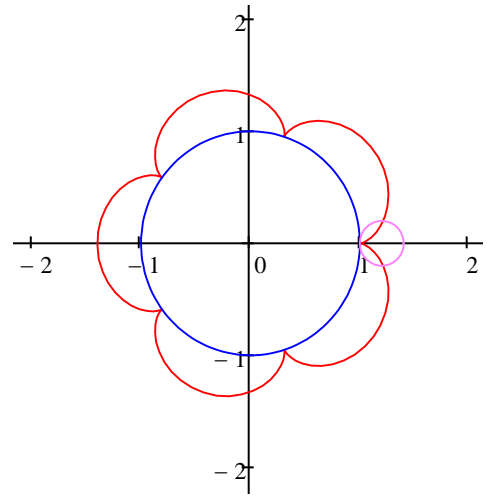
$$\begin{cases} x = (R+r)\cos t - r \cos \frac{R+r}{r}t, \\ y = (R+r)\sin t - r \sin \frac{R+r}{r}t. \end{cases}$$

— рухомий об'єкт (коло); — нерухомий об'єкт (коло).

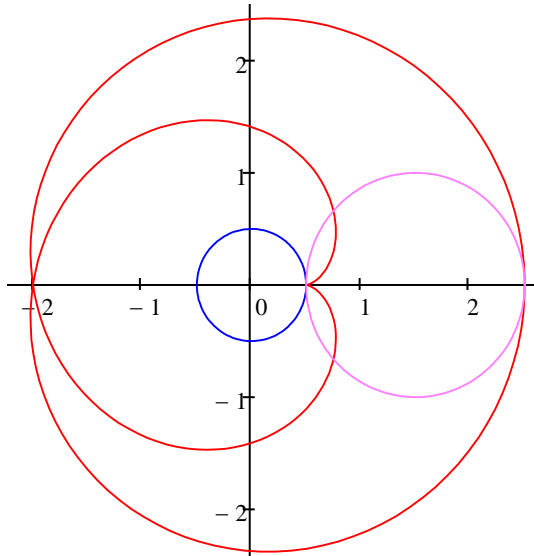
$R=1, r=1.$



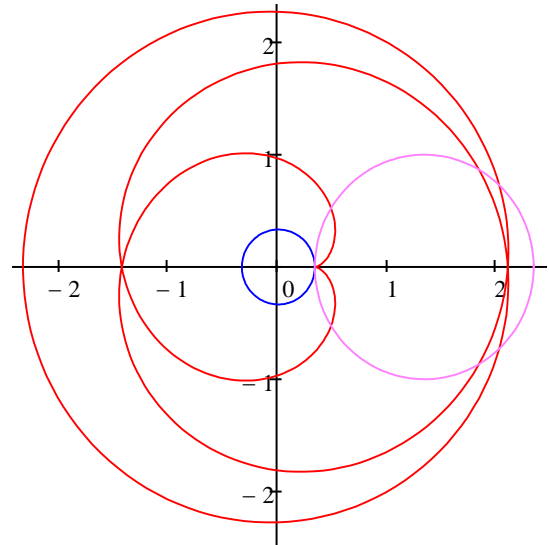
$R=1, r=1/5.$



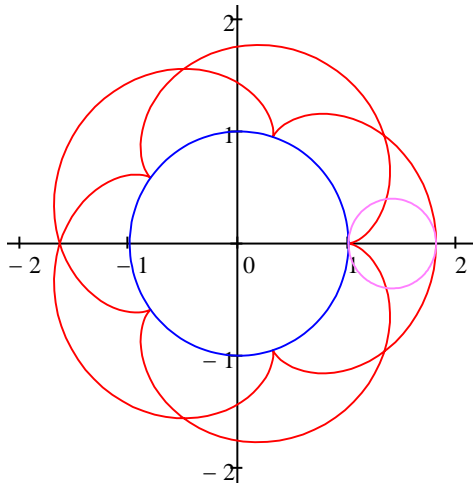
$R=1/2, r=1.$



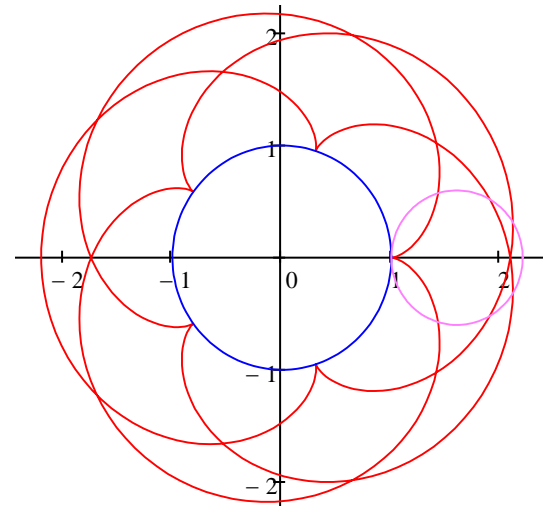
$R=1/3, r=1.$



$R=1, r=2/5.$



$R=1, r=3/5.$

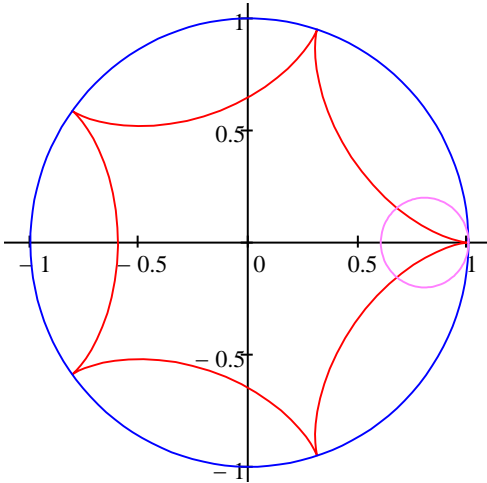


Гіпоциклоїда

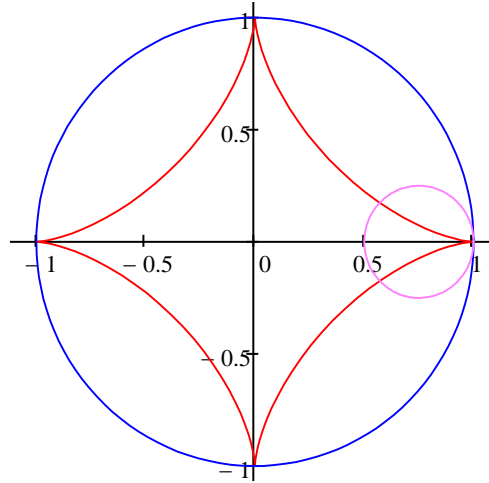
$$\begin{cases} x = (R - r) \cos t + r \cos \frac{R-r}{r} t, \\ y = (R - r) \sin t - r \sin \frac{R-r}{r} t. \end{cases}$$

— рухомий об'єкт (коло); — нерухомий об'єкт (коло).

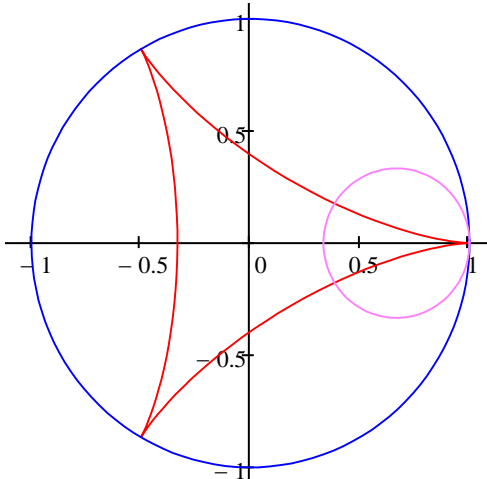
$R = 1, r = 1/5.$



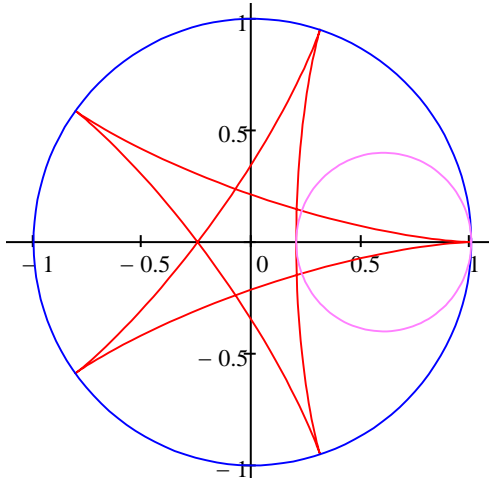
$R = 1, r = 1/4.$



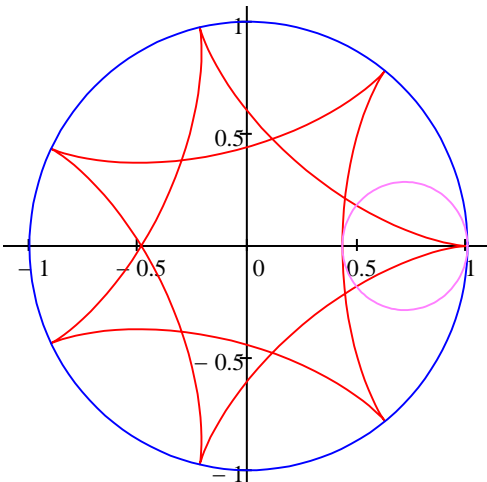
$R = 1, r = 1/3.$



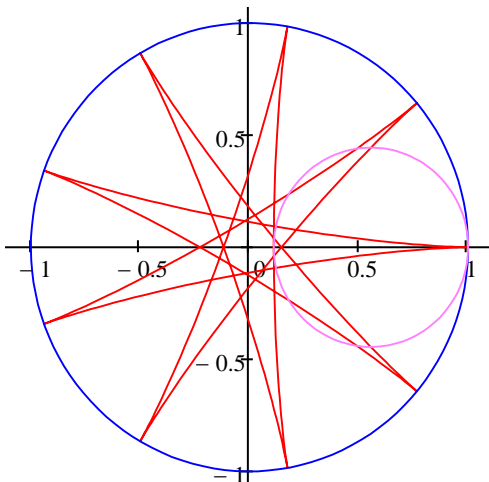
$R = 1, r = 2/5.$



$R = 1, r = 2/7.$

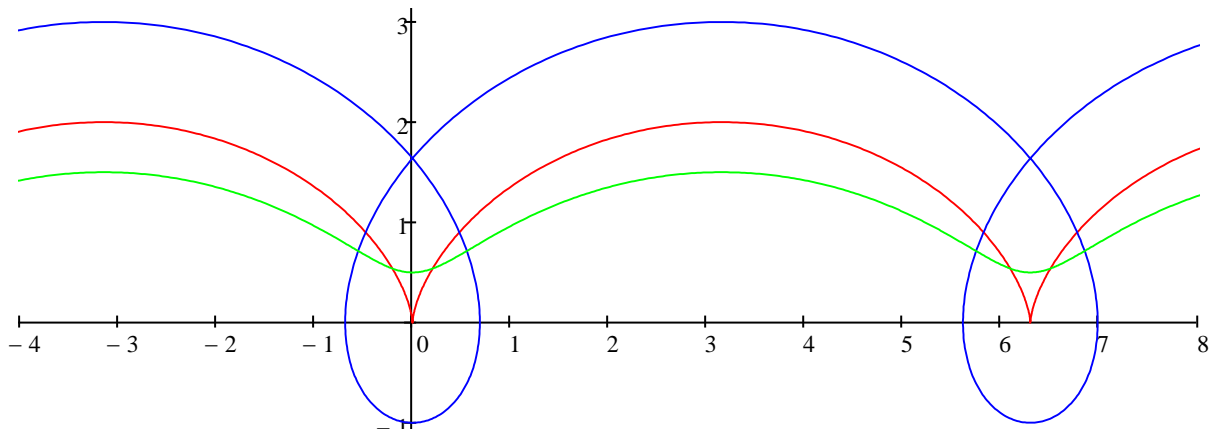


$R = 1, r = 4/9.$



Трохоїда

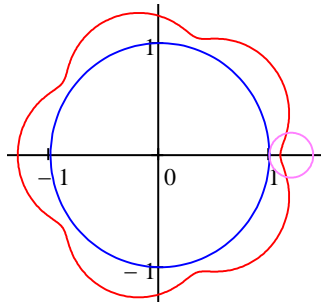
$$\begin{cases} x = rt - h \sin t, \\ y = r - h \cos t. \end{cases} \text{ при } r = 1, \text{ — } h = 1, \text{ — } h = 2; \text{ — } h = 0.5.$$



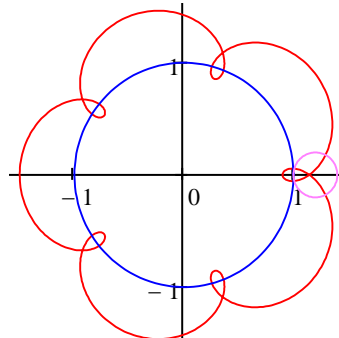
Епітрохіда

$$\begin{cases} x = (R+r) \cos t - h \cos \frac{R+r}{r} t, \\ y = (R+r) \sin t - h \sin \frac{R+r}{r} t. \end{cases} \text{ — рухомий об'єкт (коло); — нерухомий об'єкт (коло).}$$

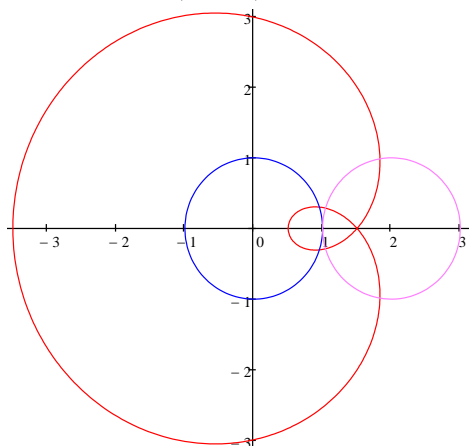
$$R = 1, r = 0.2, h = 0.1.$$



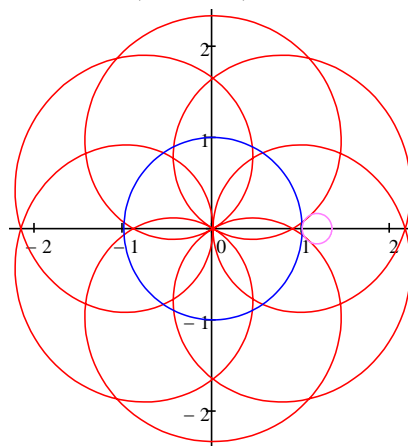
$$R = 1, r = 0.2, h = 0.3.$$



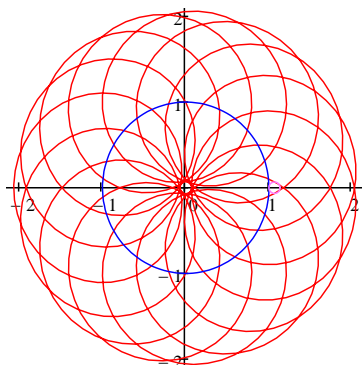
$$R = 1, r = 1, h = 1.5.$$



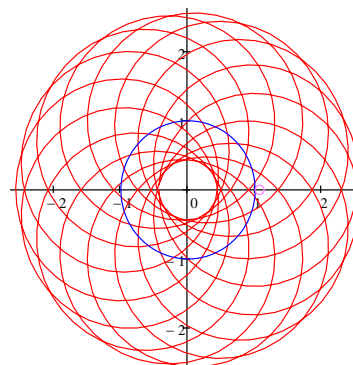
$$R = 1, r = 0.2, h = 0.3.$$



$$R = 1, r = 1/15, h = 1.$$



$$R = 1, r = 1/15, h = 1.5.$$

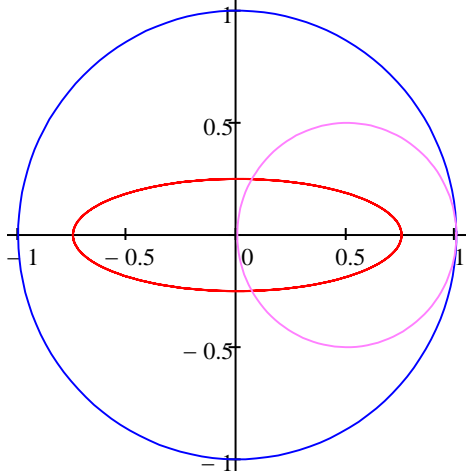


Гіпотрохіда

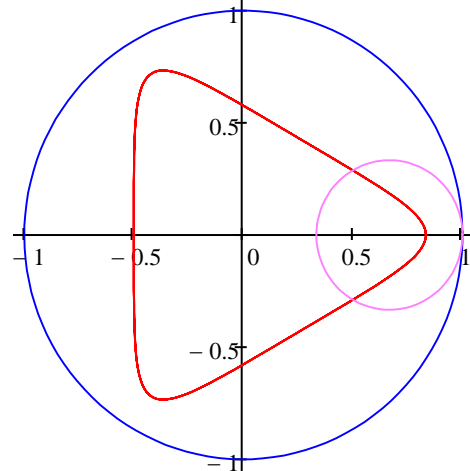
$$\begin{cases} x = (R - r) \cos t + h \cos \frac{R-r}{r} t, \\ y = (R - r) \sin t - h \sin \frac{R-r}{r} t. \end{cases}$$

— рухомий об'єкт (коло); — нерухомий об'єкт (коло).

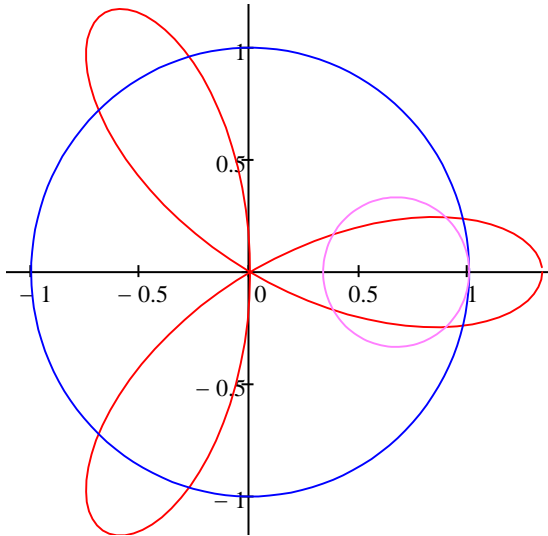
$$R = 1, r = 1/2, h = 1/4.$$



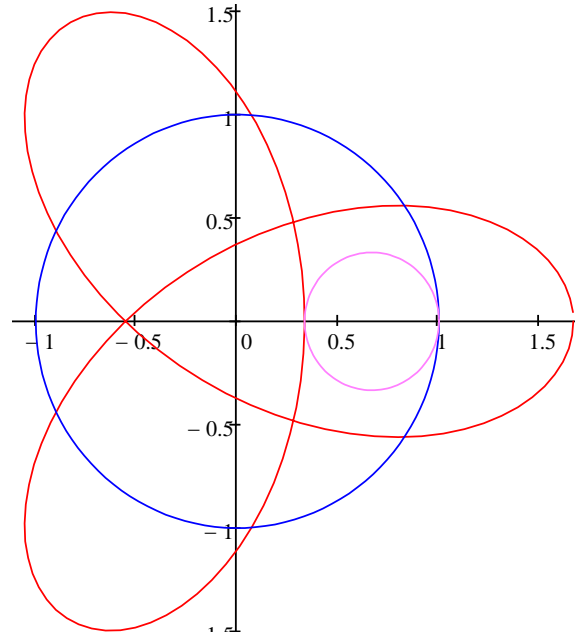
$$R = 1, r = 1/3, h = 1/6.$$



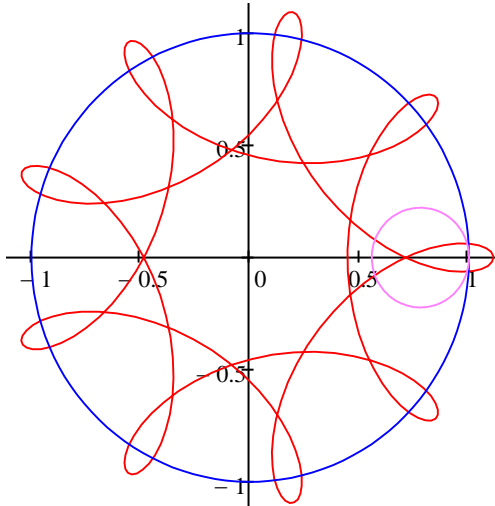
$$R = 1, r = 1/3, h = 2/3.$$



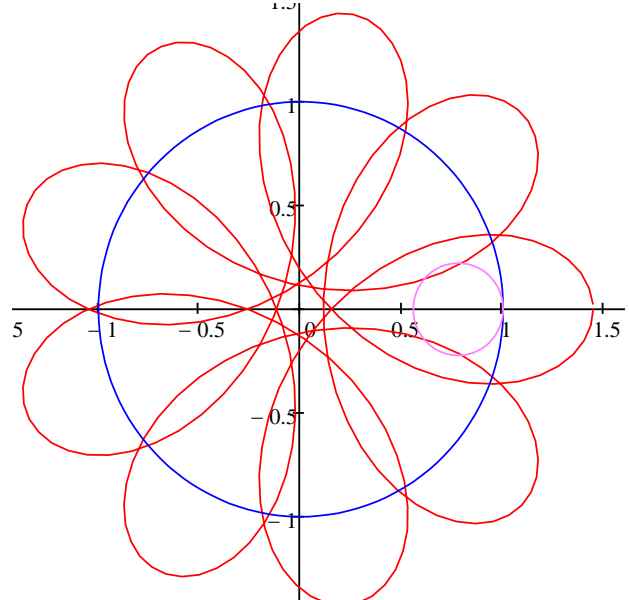
$$R = 1, r = 1/3, h = 1.$$



$$R = 1, r = 2/9, h = 1/3.$$

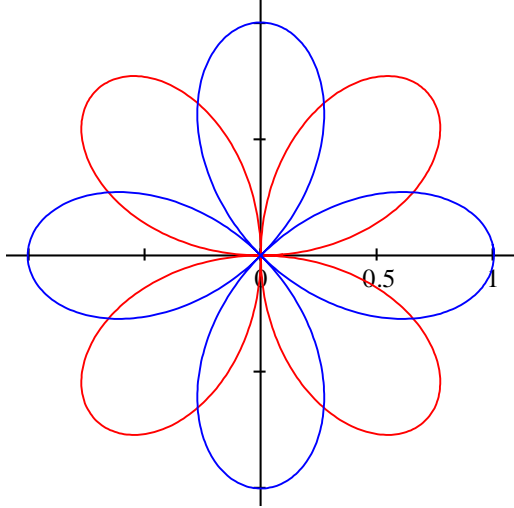


$$R = 1, r = 2/9, h = 2/3.$$



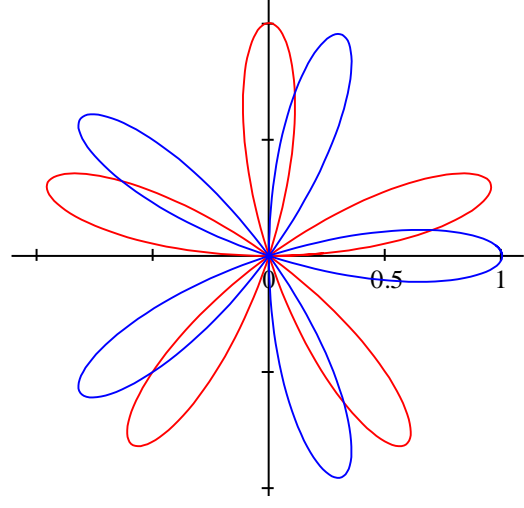
Роза

— $\rho = a \sin 2\varphi$; — $\rho = a \cos 2\varphi$

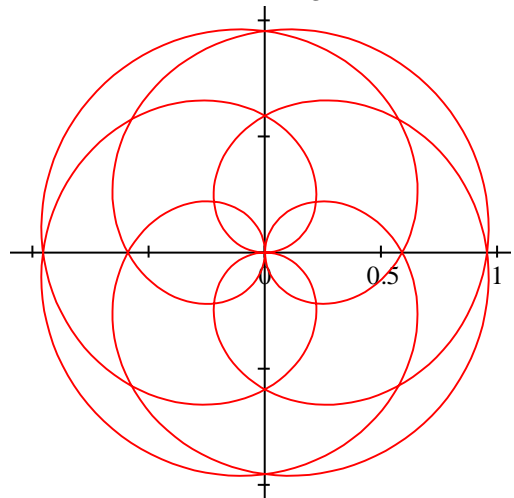


$\rho = a \sin \frac{2\varphi}{5}$

— $\rho = a \sin 5\varphi$; — $\rho = a \cos 5\varphi$

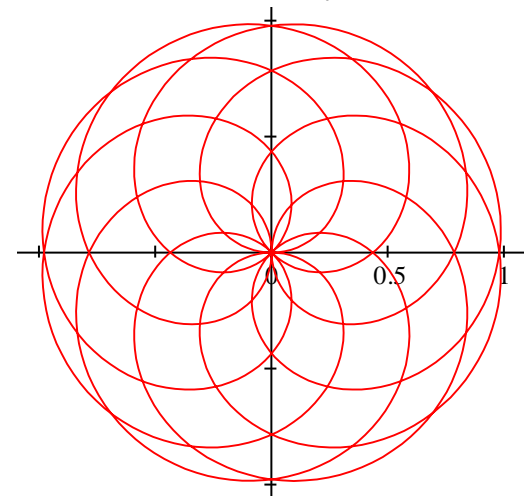
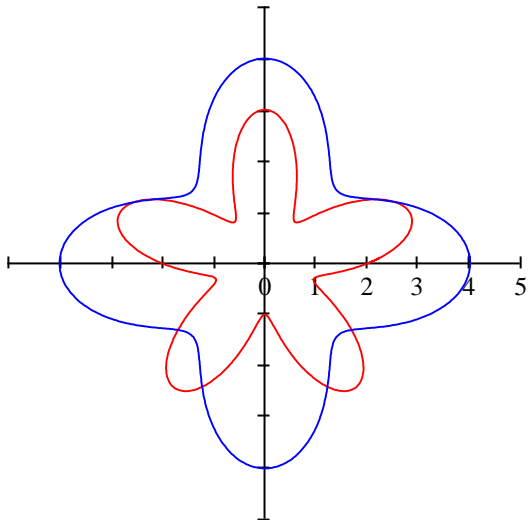


$\rho = a \sin \frac{4\varphi}{7}$



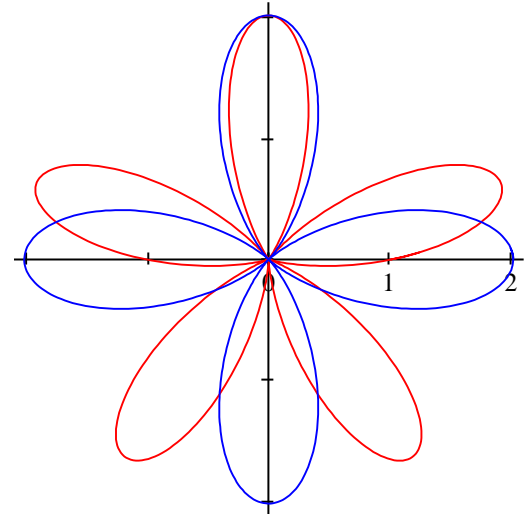
Лінії

— $\rho = 2 + \sin 5\varphi$; — $\rho = 3 + \cos 4\varphi$



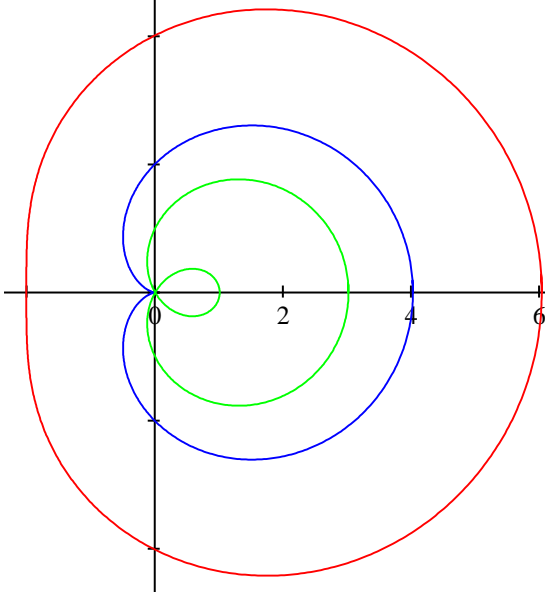
Лінії

— $\rho = 1 + \sin 5\varphi$; — $\rho = 1 + \cos 4\varphi$

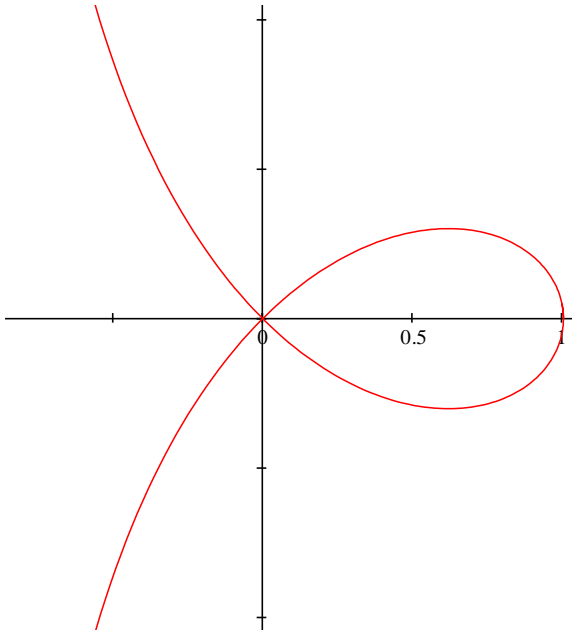


Равлик Паскаля $\rho = 2a(h + \cos \varphi)$.

При $a=1$ та $h=2$; $h=1$; $h=1/2$.



Строфоїда $\rho = \frac{\cos 2\varphi}{\cos \varphi}$.

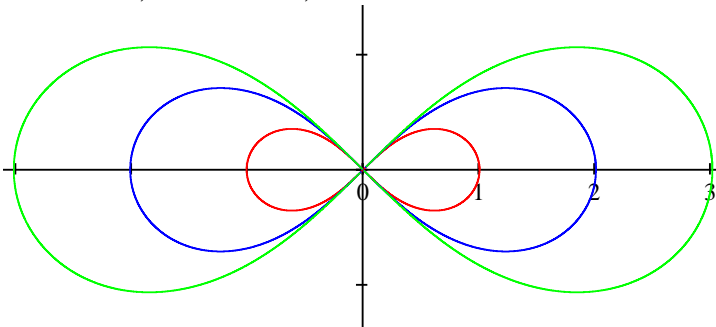


Лемніската Бернуллі

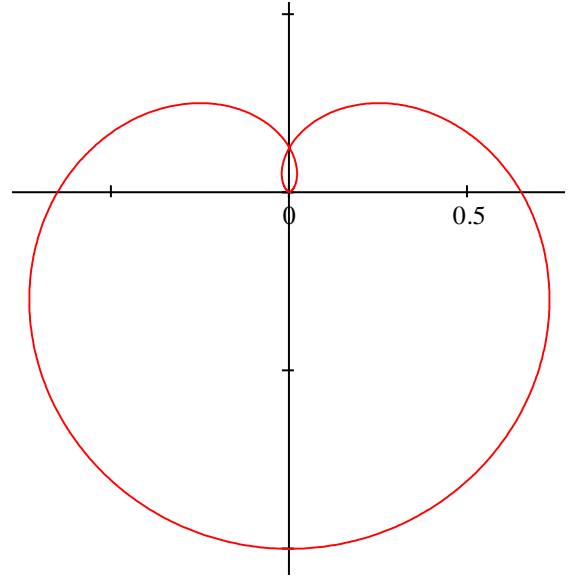
$$(x^2 + y^2)^2 = a^2(x^2 - y^2),$$

$$\rho = a\sqrt{\cos 2\varphi}.$$

$a=1$; $a=2$; $a=3$.

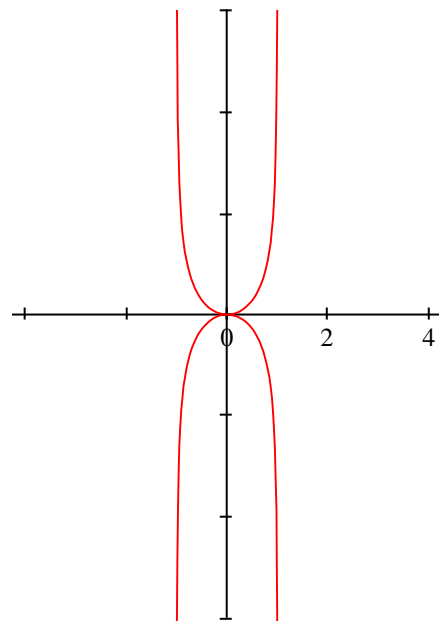


Лінія $\rho = a \sin^3 \frac{\varphi}{3}$. $a=1$.



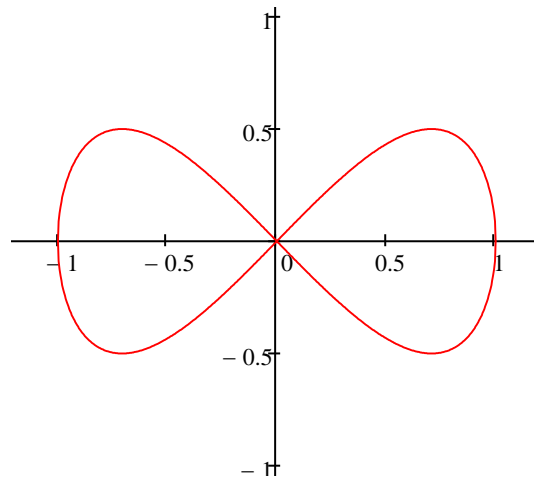
Лінія $\rho = a \operatorname{tg} \varphi$.

$a=1$.

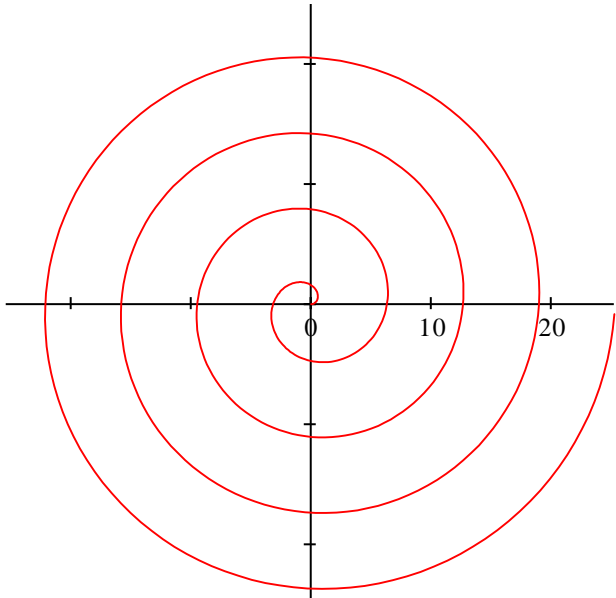


Лемніската Жероно

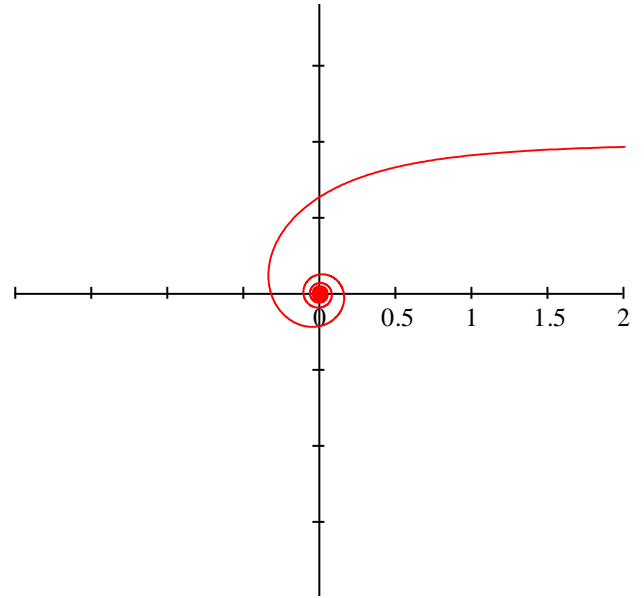
$$y^2 = x^2 - x^4$$



Спіраль Архімеда $\rho = a\varphi$. — при $a = 1$.

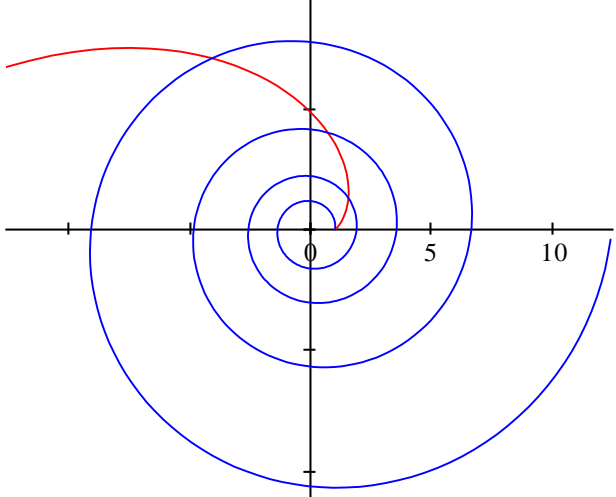


Гіперболічна спіраль $\rho\varphi = a$. — при $a = 1$.

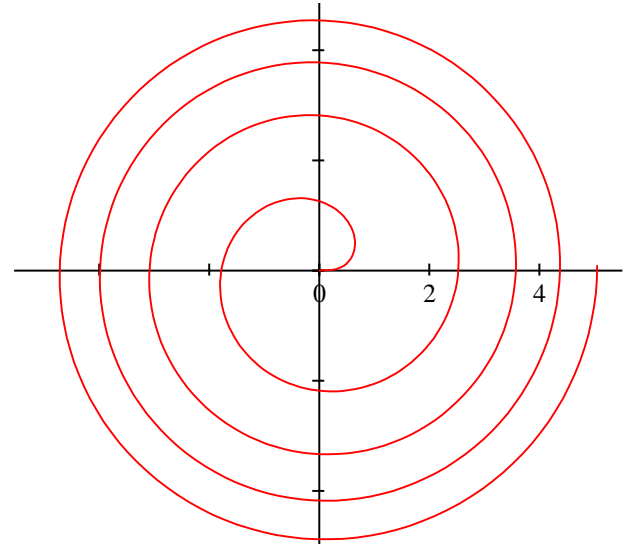


Логарифмічна спіраль $\rho = ae^{m\varphi}$

— $\rho = e^\varphi$; — $\rho = e^{0.1\varphi}$.

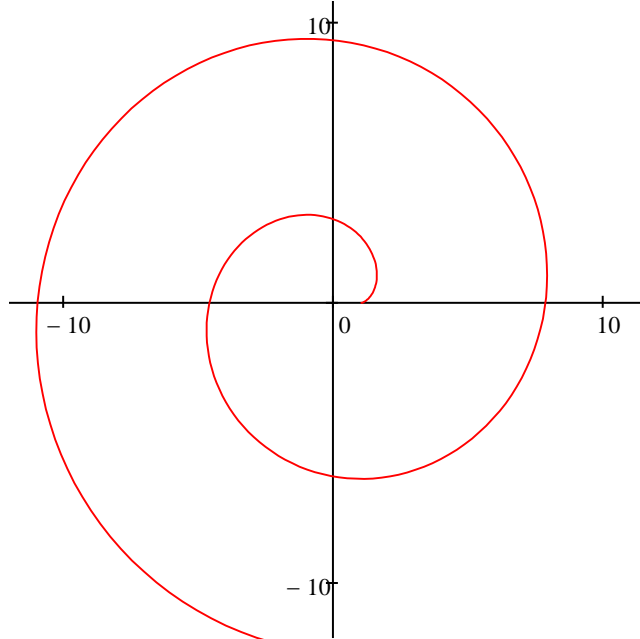


Спіраль Ферма $\rho = \sqrt{\varphi}$.



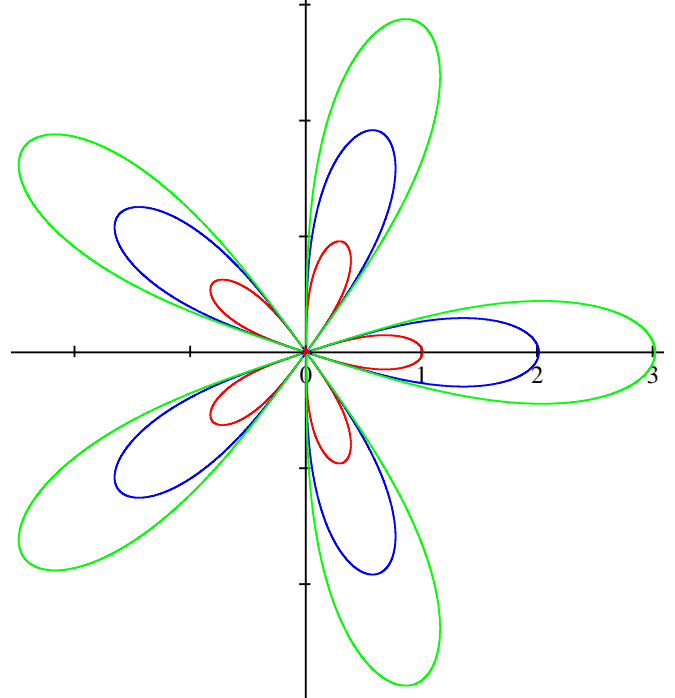
Евольвента кола

$\begin{cases} x = R(\cos t + t \sin t), \\ y = R(\sin t - t \cos t). \end{cases}$ — евольвента кола $R = 1$.



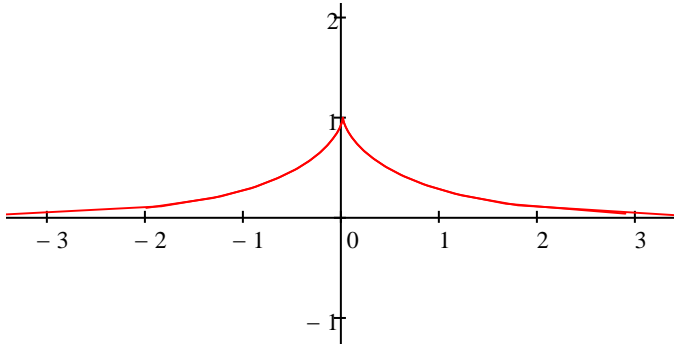
Лінія $\rho = a\sqrt{\cos 5\varphi}$.

— $a = 1$; — $a = 2$; — $a = 3$.



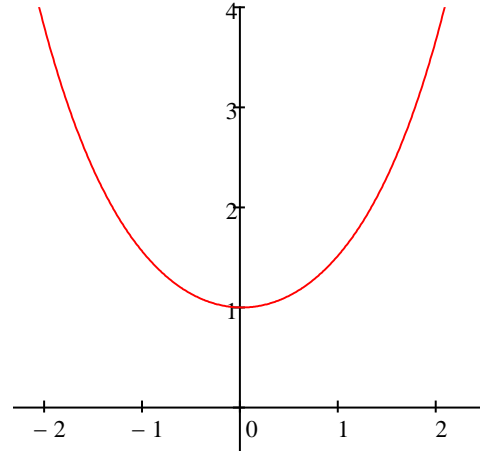
Трактриса

$$\begin{cases} x = a \left(\cos t + \operatorname{Intg} \frac{t}{2} \right), \\ y = a \sin t. \end{cases} \quad \text{— при } a = 1.$$



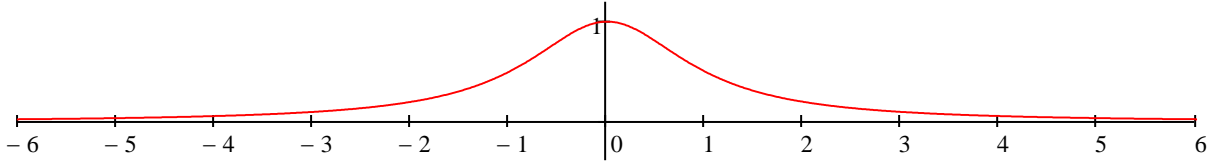
Ланцюгова лінія

$$y = a \operatorname{ch} \frac{x}{a} \quad \text{— при } a = 1.$$



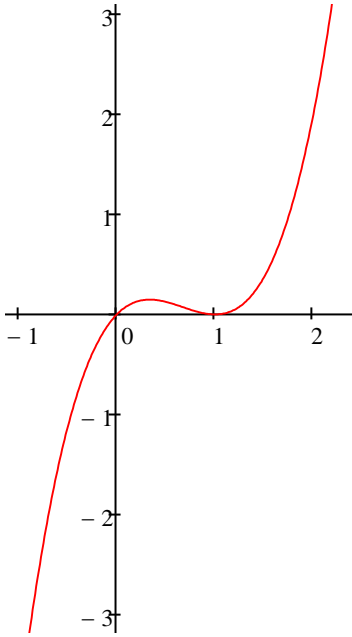
Варз'єра Ан'єзі

$$y = \frac{8a^3}{4a^2 + x^2} \quad \text{— при } a = \frac{1}{2}.$$



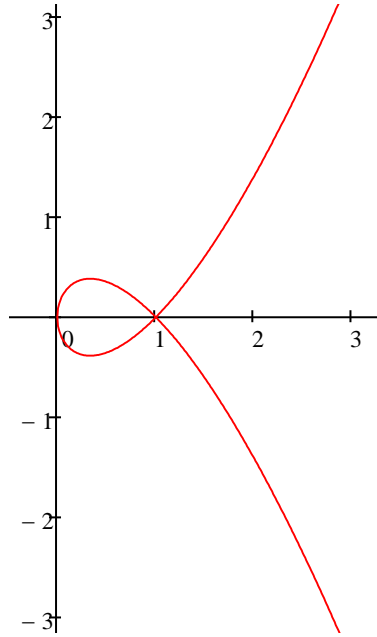
Лінія

$$y = x(x-1)^2$$



Лінія

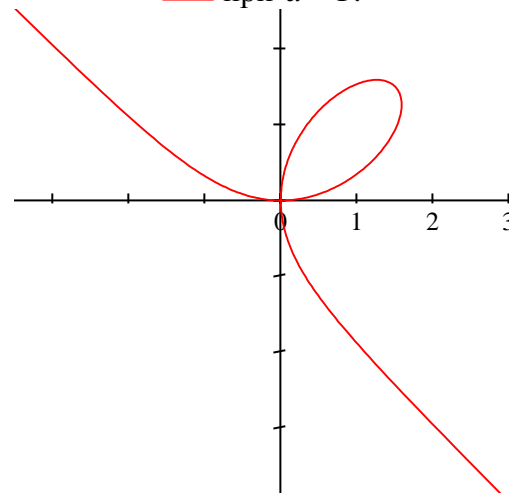
$$y^2 = x(x-1)^2$$



Лист Декарта

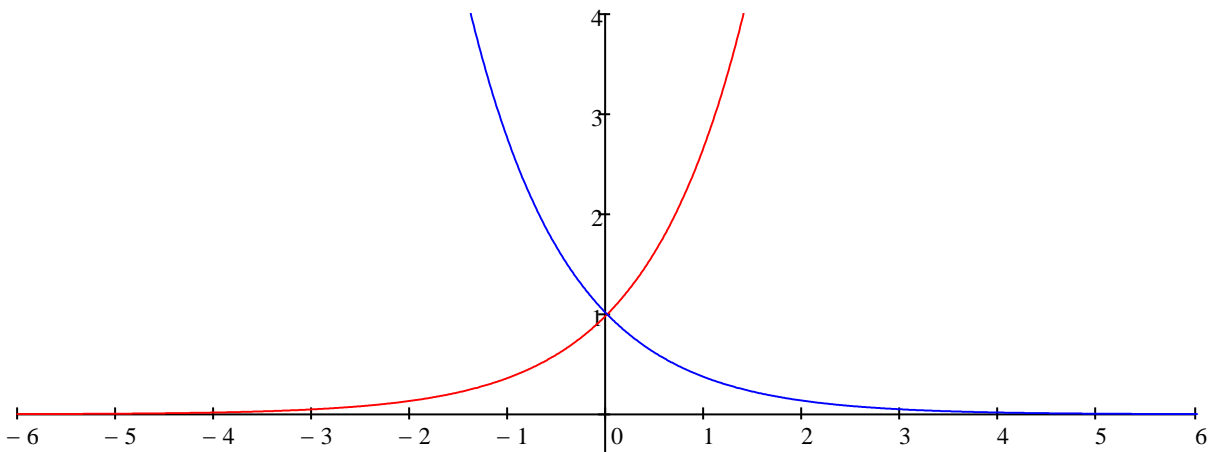
$$x^3 + y^3 = 3axy, \quad \rho = \frac{3a \cos \varphi \sin \varphi}{\cos^3 \varphi + \sin^3 \varphi}$$

— при $a = 1$.

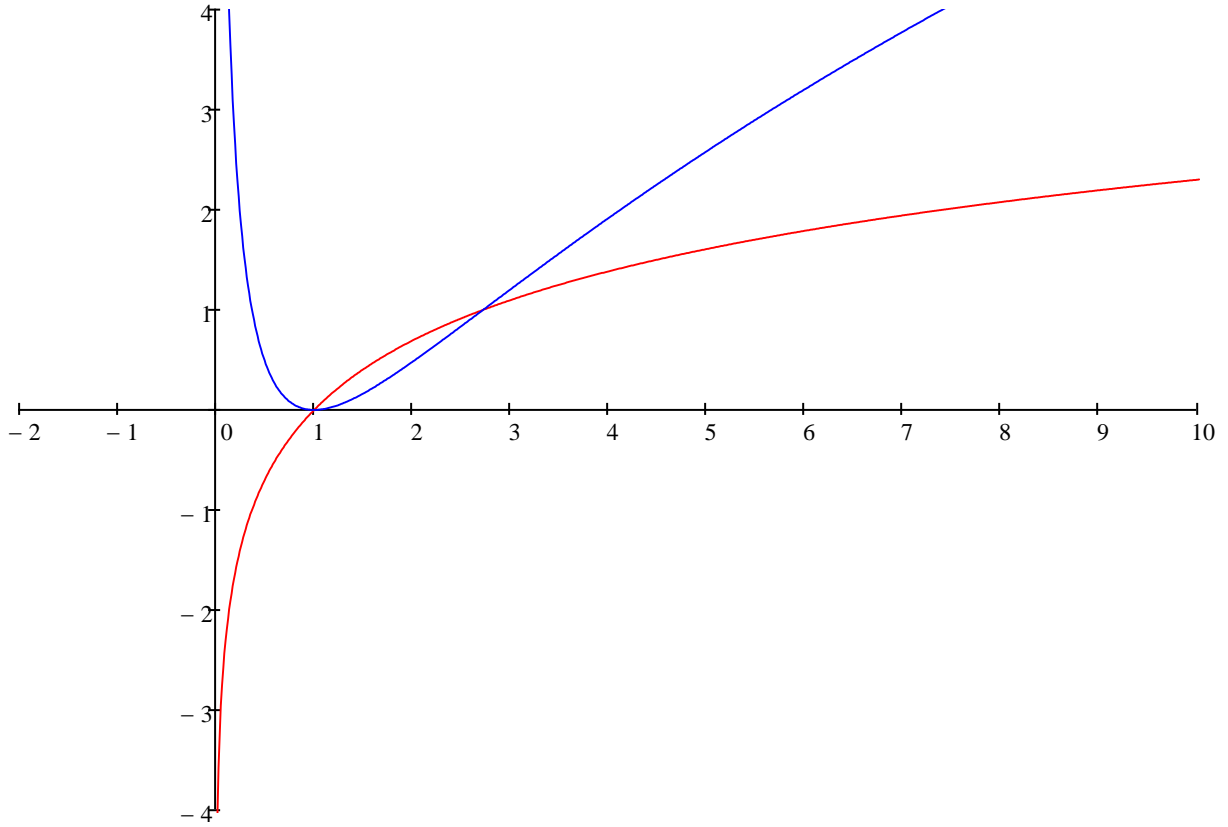


Експонента

$$\text{— } y = e^x, \quad \text{— } y = e^{-x}.$$

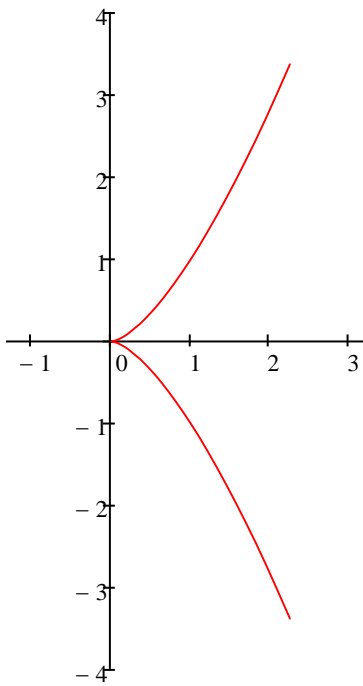


Лінії — $y = \ln x$, $y = \ln^2 x$.



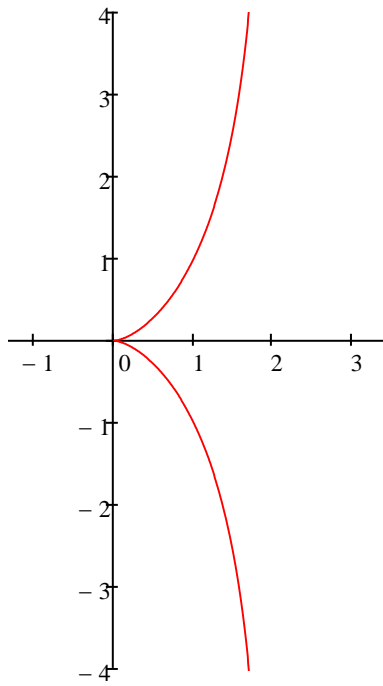
Пів кубічна парабола

$$\begin{cases} x = t^2, \\ y = at^3. \end{cases} \quad y^2 = ax^3.$$



Цисоїда

$$y^2 = \frac{x^3}{2a - x}.$$



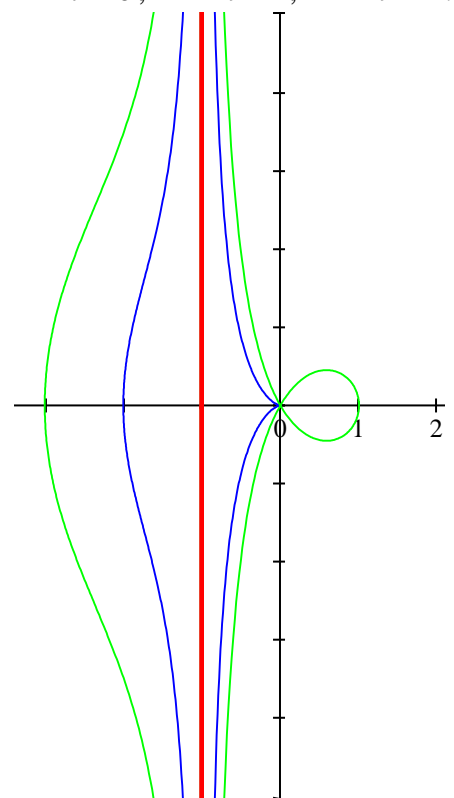
Конхоїда прямої

$$l^2 y^2 = (x^2 + y^2)(y + a)^2,$$

$$\rho = -\frac{a}{\cos \varphi} \pm l.$$

При $a=1$ та

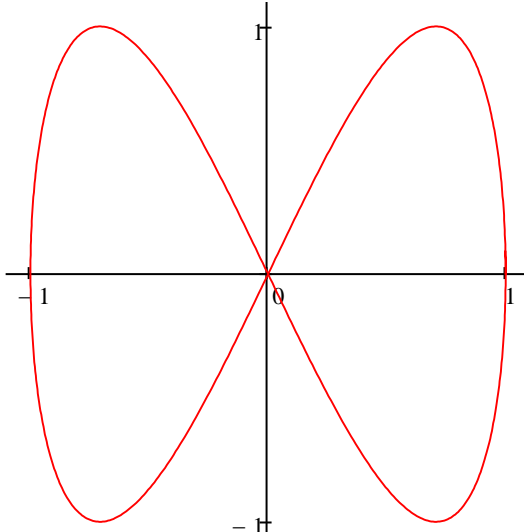
— $l=0$; — $l=1$; — $l=2$.



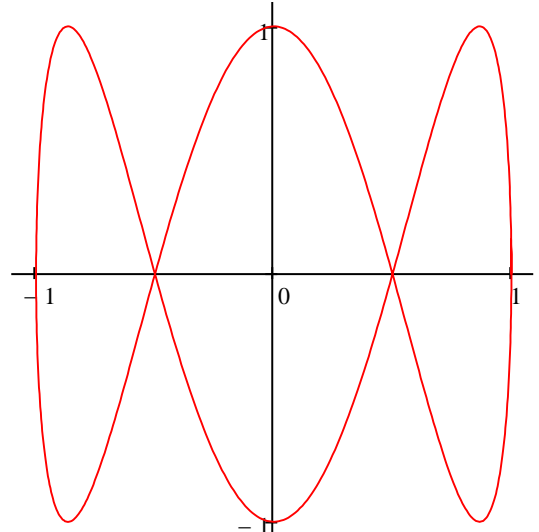
Фігури Лісажу

$$\begin{cases} x = A \cos(at + \delta), \\ y = B \sin(bt). \end{cases} \quad \text{При } A=1, B=1, \delta=0.$$

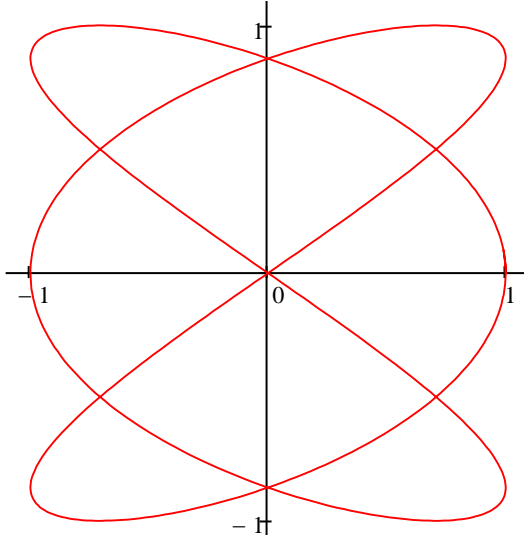
$$a=1, b=2.$$



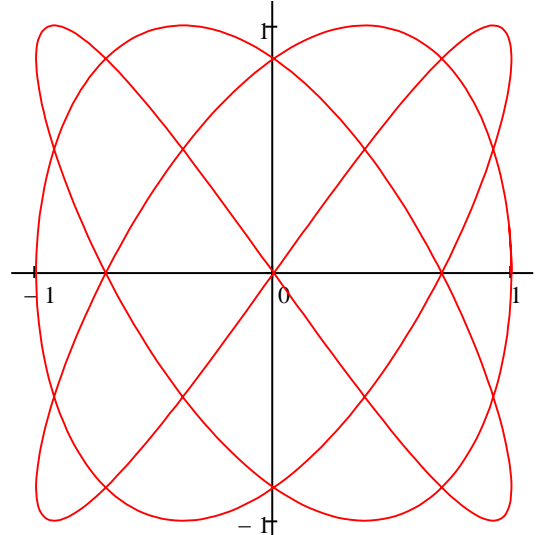
$$a=1, b=3.$$



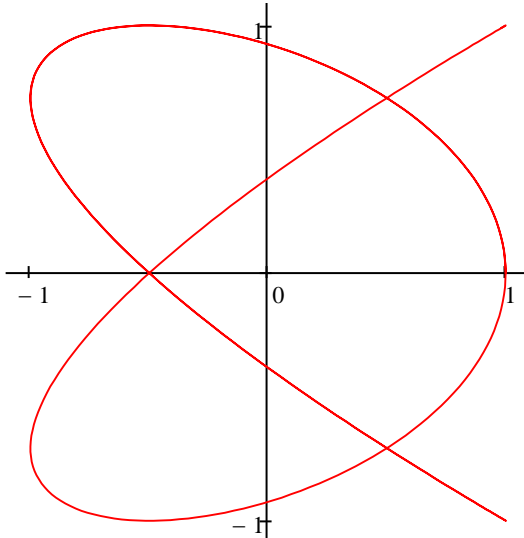
$$a=3, b=2.$$



$$a=3, b=4.$$



$$a=4, b=3.$$



$$a=5, b=9, \delta = \pi/2.$$

